

Digital Control Lec 6

Ex: $\overline{GH}(z) = \frac{K(z-0.2)}{(z-1)(z+0.6)^2}$ draw the root locus

and find the range of K for stability

1) Poles: $n_p = 3 \Rightarrow 1, -0.6, -0.6$

Zeros: $n_z = 1 \Rightarrow 0.2$

2) Z-Plane

3) Real Part

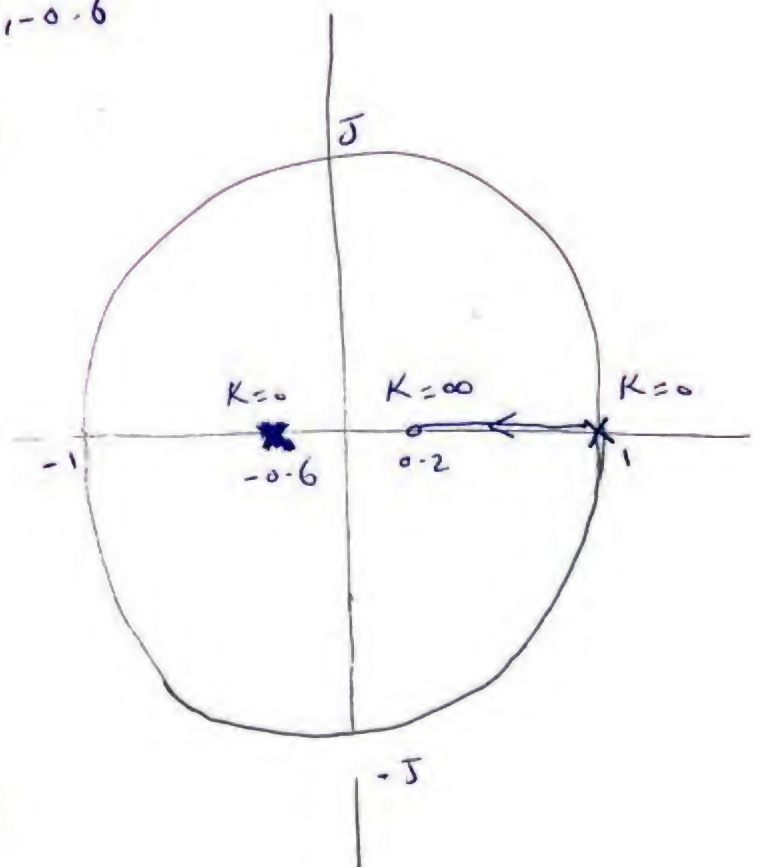
From 1 to 0.2

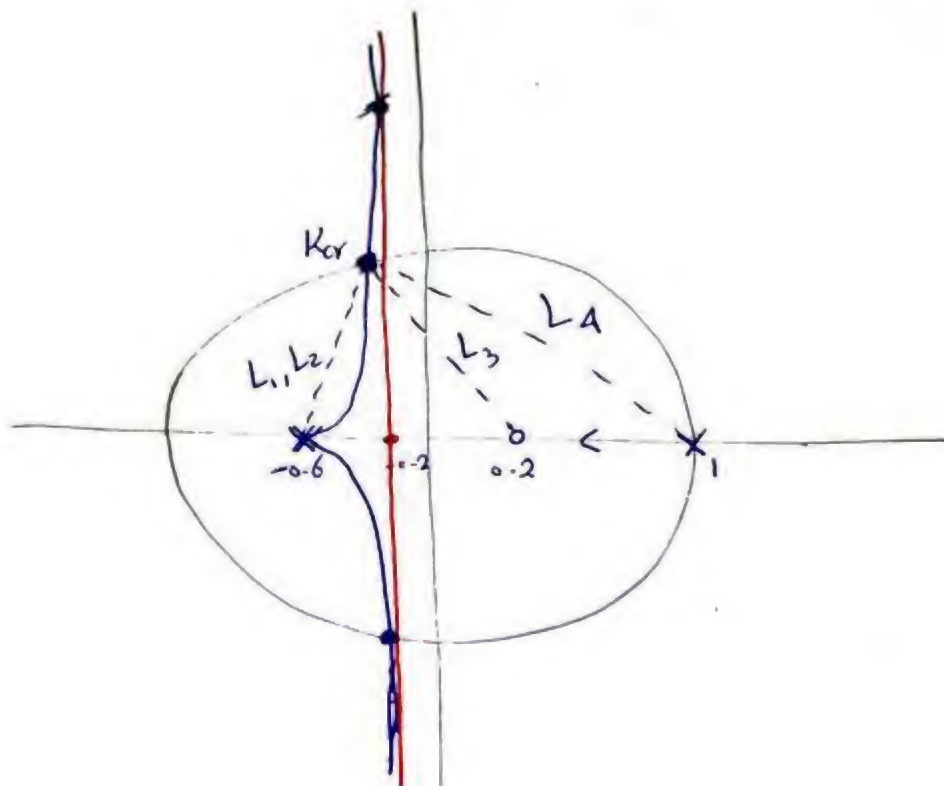
④ Asymptotes

no. of Asym. = $n_p - n_z = 2$

$$C_A = \frac{\sum \text{Poles} - \sum \text{zeros}}{n_p - n_z} = \frac{(1 - 0.6 - 0.6) - (0.2)}{2} = -0.2$$

$$\Theta = \frac{(2L + 1)180}{n_p - n_z} \begin{cases} \Theta_1 = 90^\circ \\ \Theta_2 = -90^\circ \end{cases}$$





[5] Determine K_{cr} :

$$K_{cr} = \frac{\prod \text{Poles}}{\prod \text{Zeros}} \times \frac{L_1 \times L_2 \times L_4}{L_3}$$

another solution \Rightarrow use bilinear transformation

$$z = \frac{1+y}{1-y}$$

$$\text{ch. equation: } 1 + G H(z) = 0$$

$$1 + \frac{K(z-0.2)}{(z-1)(z-0.6)} = 0$$

$$(z-1)(z-0.6) + K(z-0.2) = 0$$

$$z = \frac{1+y}{1-y}$$

$$(-0.32 + 1.2K)$$

$$(0.32 + 1.2K)y^3 + (2.56 - 1.6K)y^2 + (5.12 - 0.4K)y$$

$$+ 0.8K = 0$$

$$\begin{array}{l|l} y^3 & \underline{0.32 + 1.2K} > 0 & 5.12 - 0.4K \\ y^2 & \underline{2.56 - 1.6K} > 0 & 0.8K \\ y^1 & \underline{A} > 0 & \\ y^0 & \underline{0.8K} > 0 & \end{array}$$

الشروط التي تحققها جذرين يسوي
نقاطها هو range of K

$$1) 0.32 + 1.2K > 0 \Rightarrow K > -0.267$$

$$2) 2.56 - 1.6K > 0 \Rightarrow K < 1.6$$

$$3) 0.8K > 0 \Rightarrow K > 0$$

$$4) A > 0$$

$$\left[(2.56 - 1.6K)(5.12 - 0.4K) \right] - 0.8K(0.32 + 1.2K) > 0$$

$$-0.32K^2 - 9.472K + 13.107 > 0$$

$$K^2 + 29.6K - 40.96 < 0$$

$$K_{1,2} = 1.32 \text{ \& } -30.92$$

$$(K - 1.32)(K + 30.92) < 0$$

قوسين أعلى من الصفر كما أحدهما موجب والآخر سالب إذا انعكس.

$$(K - 1.32)(K + 30.92) < 0$$

$$\begin{array}{cc} + & - \\ K > 1.32 & K < -30.92 \end{array} \Rightarrow X$$

$$\begin{array}{cc} - & + \\ K < 1.32 & K > -30.92 \end{array} \quad \checkmark \quad (4)$$

The range of K for stability

$$0 < K < 1.32$$

Bode diagram

→ To study system Properties in Freq. domain
For a discrete time system, we use bilinear
Transformation, to get the system in
Cont. time domain.

$$\text{bilinear Transformation} \Rightarrow \boxed{Z = \frac{1+s}{1-s}}$$

* Bode diagram is one of relative stability methods.

→ Given open loop Digital T.F $\overline{GH}(z)$,

To Draw the bode diagram.

1) Map From Z-domain to r-domain.

$$Z = \frac{1+r}{1-r}$$

$$\overline{GH}(z) \longrightarrow \overline{GH}(r)$$

$$2) r \longrightarrow j\omega_r$$

$$\overline{GH}(r) \longrightarrow \overline{GH}(j\omega_r)$$

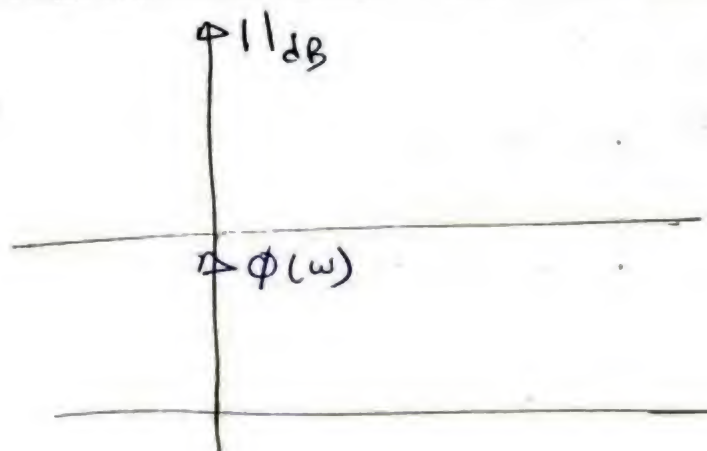
$$3) |\overline{GH}(j\omega_r)| = \frac{|\text{البسط}|}{|\text{المقام}|}$$

$$|\overline{GH}(j\omega_r)|_{dB} = 20 \log |\overline{GH}(j\omega_r)|$$

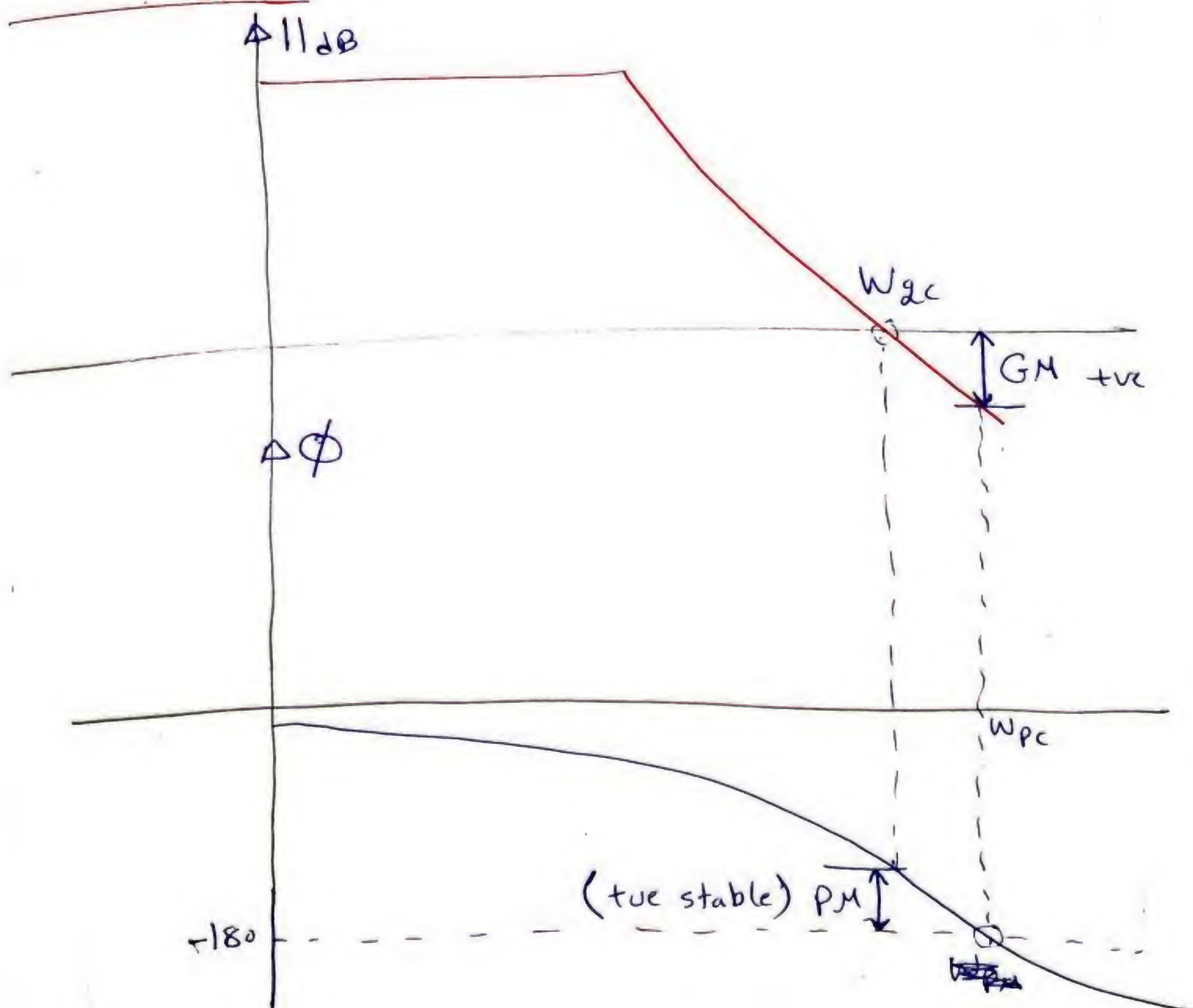
$$4) \phi(\omega_r) = \angle \text{البسط} - \angle \text{المقام}$$

$$= \tan^{-1} \left(\frac{\text{Imag.}}{\text{Real}} \right)$$

⑤ Draw

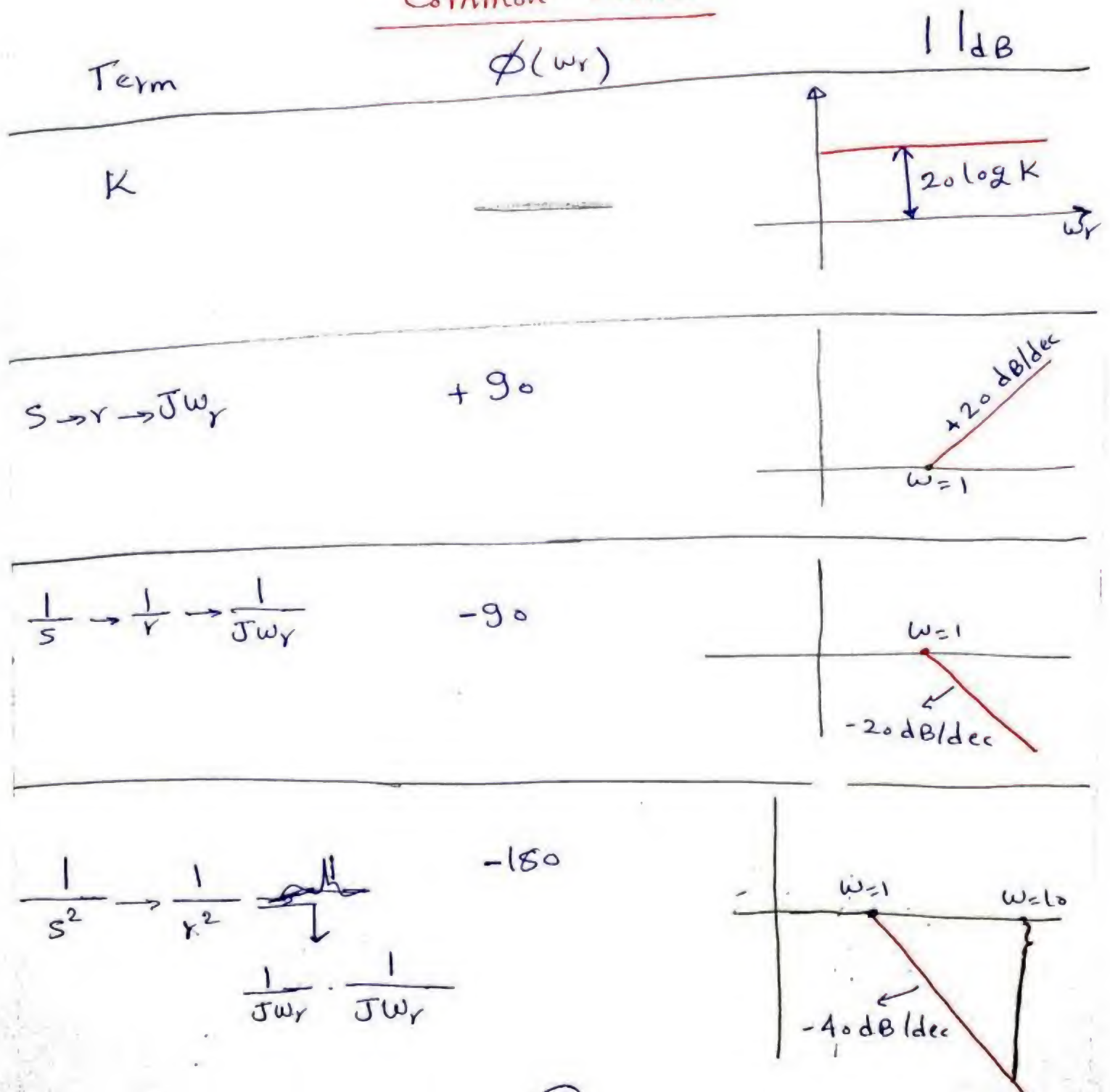


For example



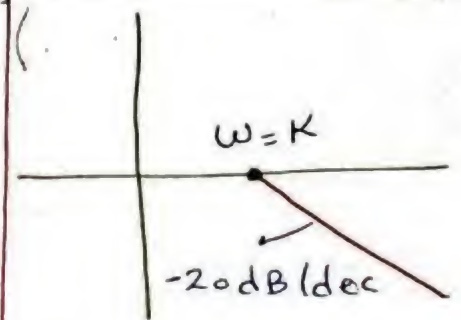
GM $\begin{cases} > 0 & \text{(stable)} \\ < 0 & \text{(unstable)} \\ = 0 & \text{(critically stable)} \end{cases}$

Common terms



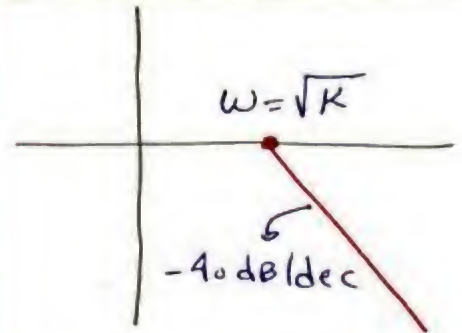
$$\frac{K}{s} \rightarrow \frac{K}{r} \rightarrow \frac{K}{s\omega_r}$$

-90



$$\frac{K}{s^2} \rightarrow \frac{K}{r^2}$$

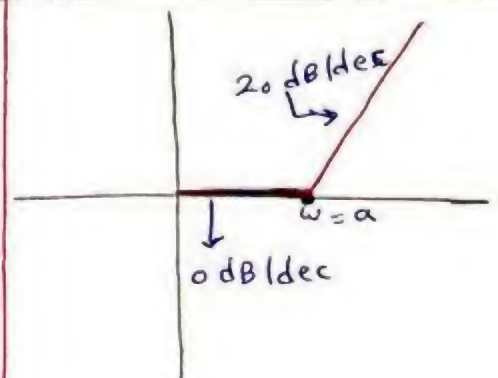
-180



$$\left(1 + \frac{s}{a}\right) \rightarrow 1 + \frac{r}{a}$$

$$\hookrightarrow 1 + j\frac{\omega_r}{a}$$

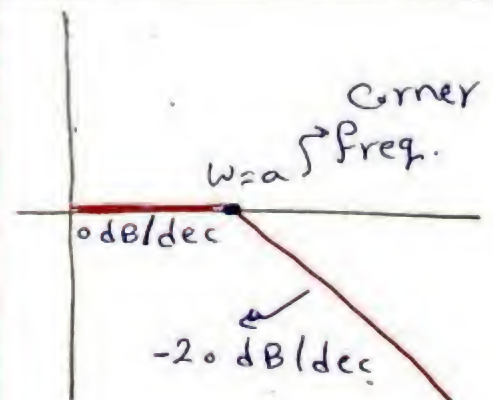
$\tan^{-1}\left(\frac{\omega_r}{a}\right)$



$$\frac{1}{1 + \frac{s}{a}} \rightarrow \frac{1}{1 + \frac{r}{a}}$$

$$\frac{1}{1 + j\frac{\omega_r}{a}}$$

$-\tan^{-1}\left(\frac{\omega_r}{a}\right)$



EX 1 Draw Bode diagram for the following system:-

$$\overline{GH}(z) = \frac{0.5 (z + 0.76)}{(z - 1)(z - 0.45)}$$

and find ω_{gc} & ω_{pc} & GM & PM.

$$1) z = \frac{1+r}{1-r}$$

$$GH(r) = \frac{0.5 \left(\frac{1+r}{1-r} + 0.76 \right)}{\left(\frac{1+r}{1-r} - 1 \right) \left(\frac{1+r}{1-r} - 0.45 \right)}$$

$$= \frac{0.5(1-r)(1+r + 0.76(1-r))}{(1+r - 1(1-r))(1+r - 0.45(1-r))}$$

$$= \frac{0.5(1-r)(1.76 + 0.24r)}{2r(0.55 + 1.45r)}$$

$$GH(r) = \frac{0.5(1-r) * 1.76 \left(1 + \frac{r}{7.33} \right)}{2(0.5)r \left(1 + \frac{r}{0.38} \right)}$$

$$\overline{GH}(s) = \frac{0.8 (1-s) \left(1 + \frac{s}{7.33}\right)}{s \left(1 + \frac{s}{0.38}\right)}$$

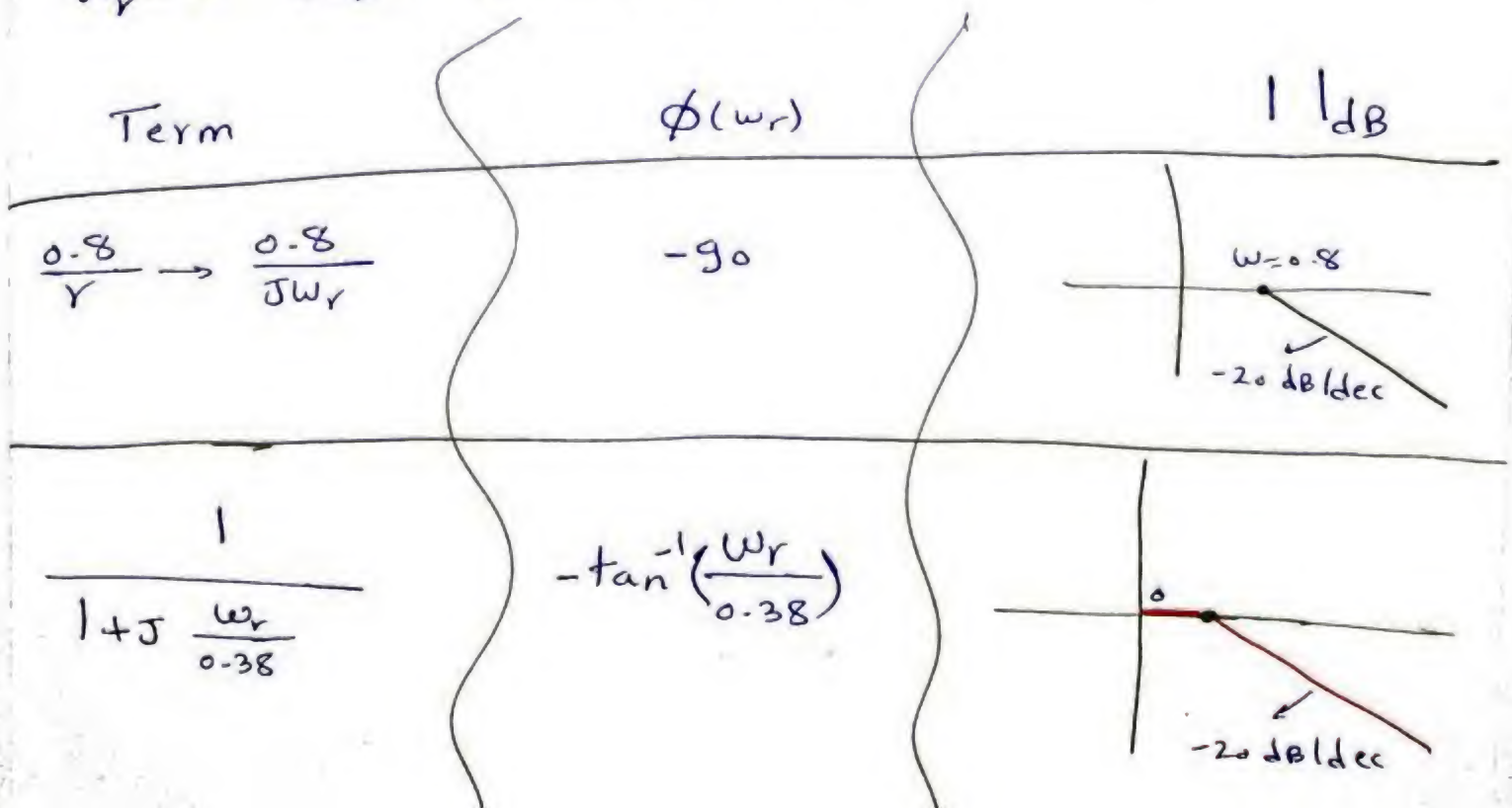
$$s \rightarrow j\omega_r$$

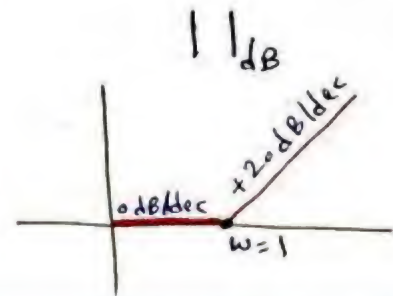
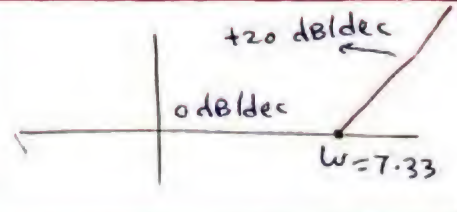
$$GH(j\omega_r) = \frac{0.8}{j\omega_r} \cdot \frac{(1 - j\omega_r) \left(1 + j \frac{\omega_r}{7.33}\right)}{\left(1 + j \frac{\omega_r}{0.38}\right)}$$

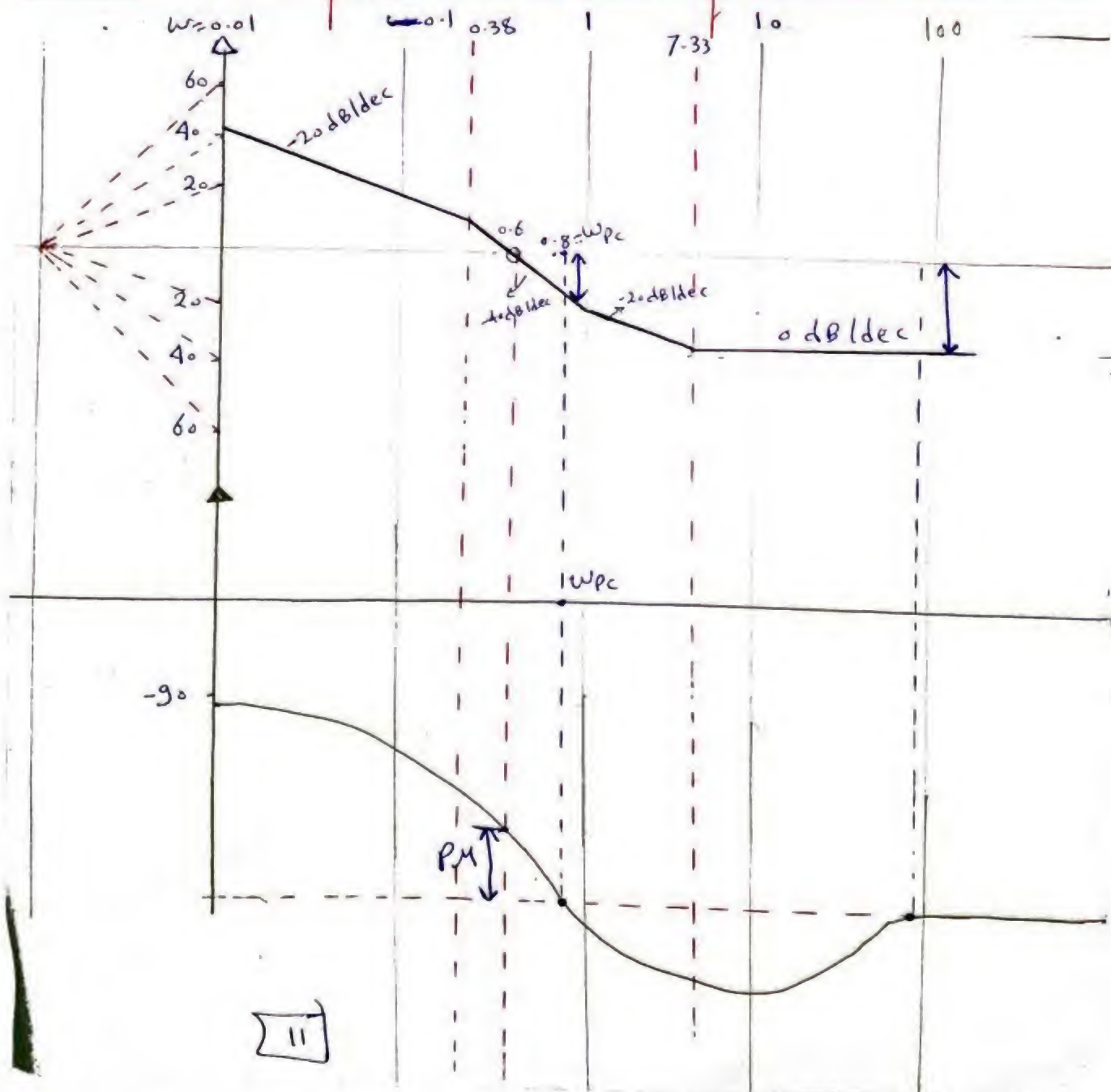
$$\phi(\omega_r) = -\tan^{-1}(\omega_r) + \tan^{-1}\left(\frac{\omega_r}{7.33}\right) - 90^\circ - \tan^{-1}\left(\frac{\omega_r}{0.38}\right)$$

$$\omega_r = 0 \Rightarrow \phi(\omega_r) = -90^\circ$$

$$\omega_r = \infty \Rightarrow \phi(\omega_r) = -180^\circ$$



Term	$\phi(\omega)$	
$1 - j \frac{\omega_r}{1}$	$-\tan^{-1}(\omega_r)$	
$1 + j \frac{\omega_r}{7.33}$	$\tan^{-1} \frac{\omega_r}{7.33}$	



ω_r	0	0.01	0.1	0.38	0.6	1
ϕ	-90°	-92°	-109.67°	-152.8°	-173.94°	-196.43°

ω_r	7.33	100	∞
ϕ	-209.47°	-183.4°	-180°

مع لو عندك قيمتين لـ GM تأخذ أسوأهما.
 له معنى القيمة الأصغر.

(12)